

Listing strategies N5

Product rule for counting:
→ $4 \times 3 \times 2 \times 1 = 24$ ways to arrange the letters P, I, X and L.

Powers and roots N6, N7

Special indices: for any value a :

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^{\left(\frac{p}{q}\right)} = \sqrt[q]{a^p}$$

→ $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$

→ $8^{\left(\frac{2}{3}\right)} = \sqrt[3]{8^2} = 4$

Surds N8

Look for the biggest square number factor of the number:

→ $\sqrt{80} = \sqrt{16 \times 5} = 4\sqrt{5}$

Rationalise the denominator N8

Multiply the numerator and denominator by an expression that makes the denominator an integer:

→ $\frac{4}{\sqrt{7}} = \frac{4 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{4\sqrt{7}}{7}$

→ $\frac{2}{4 + \sqrt{5}}$

$= \frac{2}{4 - \sqrt{5}} \times \frac{4 - \sqrt{5}}{4 - \sqrt{5}} = \frac{2(4 - \sqrt{5})}{11}$

Standard form N9

Standard form numbers are of the form $a \times 10^n$, where $1 \leq a < 10$ and n is an integer.

Recurring decimals N10

Make a recurring decimal a fraction:

→ $n = 0.23\bar{6}$

(two digits are in the recurring pattern, so multiply by 100)

$100n = 23.6\bar{6}$

(this is the same as $23.6\bar{36}$)

$99n = 23.6\bar{36} - 0.23\bar{6} = 23.4$

$n = \frac{23.4}{99} = \frac{234}{990} = \frac{13}{55}$

Error intervals N15

Find the range of numbers that will round to a given value:

→ $x = 5.83$ (2 decimal places)

$5.825 \leq x < 5.835$

→ $y = 46$ (2 significant figures)

$45.5 \leq y < 46.5$

Note use of \leq and $<$, and that the last significant figure of each is 5.

Equations and identities A3

An equation is true for some particular value of x ...

→ $2x + 1 = 7$ is true if $x = 3$

...but an identity is true for every value of x

→ $(x + a)^2 \equiv x^2 + 2ax + a^2$

(note the use of the symbol \equiv)

Laws of indices A4

For any value a :

$$a^x \times a^y = a^{x+y}$$

$$\frac{a^x}{a^y} = a^{x-y}$$

$$(a^x)^y = a^{xy}$$

→ $\left(\frac{2pq^4}{p^3q}\right)^3 = \frac{8p^3q^{12}}{p^9q^3} = \frac{8q^9}{p^6}$ or $8q^9p^{-6}$

Difference of two squares A4

$$a^2 - b^2 = (a + b)(a - b)$$

→ $x^2 - 25 = (x + 5)(x - 5)$

Rearrange a formula A5

The subject of a formula is the term on its own. Rearrange to

→ Make x the subject of

$$2x + ay = y - bx$$

$$2x + bx = y - ay$$

$$x(2 + b) = y - ay$$

$$x = \frac{y - ay}{2 + b}$$

Functions A7

Combining functions:

$$fg(x) = f(g(x))$$

→ If $f(x) = x + 3$ and $g(x) = x^2$

$$fg(x) = x^2 + 3$$

$$gf(x) = (x + 3)^2$$

The inverse of f is f^{-1}

→ If $f(x) = 2x + 5$ then

$$f^{-1}(x) = \frac{x - 5}{2}$$

$y = mx + c$ A9

Equation of straight line $y = mx + c$ m is the gradient; c is the y intercept:

→ Find the equation of the line that joins $(0, 3)$ to $(2, 11)$

Find its gradient...

$$\frac{11 - 3}{2 - 0} = \frac{8}{2} = 4$$

...and its y intercept...

Passes through $(0, 3)$, so $c = 3$.

Equation is $y = 4x + 3$.

Parallel lines: gradients are equal; perpendicular lines: gradients are "negative reciprocals".

→ $y = 2x + 3$ and $y = 2x - 5$ are parallel to each other; $y = 2x + 3$

and $y = -\frac{1}{2}x + 3$ are perpendicular

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Transformations of curves A13

Starting with the curve $y = f(x)$:

Translate $\begin{pmatrix} 0 \\ a \end{pmatrix}$ for $y = f(x) + a$

Translate $\begin{pmatrix} -a \\ 0 \end{pmatrix}$ for $y = f(x + a)$

Reflect in x axis for $y = -f(x)$

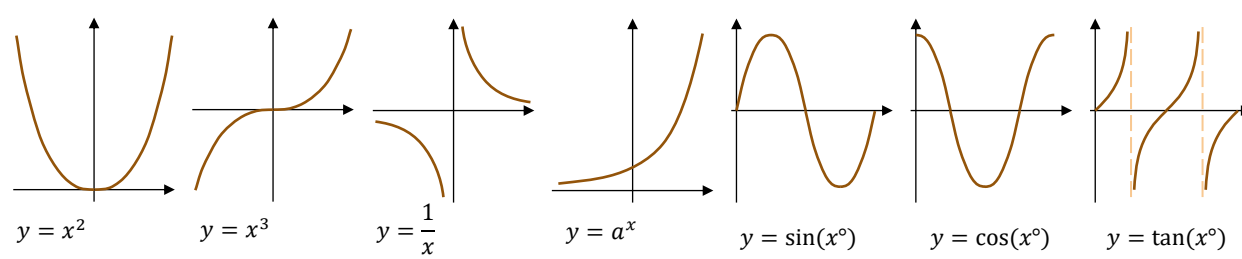
Reflect y axis for $y = f(-x)$

Velocity - time graph A15

Gradient = acceleration (you may need to draw a tangent to the curve at a point to find the gradient);

Area under curve = distance travelled.

Standard graphs A12



Quadratics A11, A18

If a quadratic equation cannot be factorised, use the formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

→ Solve $2x^2 + 3x - 7 = 0$

$$x = \frac{-3 \pm \sqrt{9 - (-56)}}{2 \times 2} = -2.73$$

or $x = \frac{-3 + \sqrt{9 - (-56)}}{2 \times 2} = 1.23$

Equation of a circle A16

Complete the square to find the turning point of a quadratic graph.

→ $y = x^2 - 6x + 2$

$$y = (x - 3)^2 - 9 + 2$$

$$y = (x - 3)^2 - 7$$

Turning point is at $(3, -7)$

Simultaneous equations A19

One linear, one quadratic;

→ Solve $\begin{cases} x + 3y = 10 \\ x^2 + y^2 = 20 \end{cases}$

Rearrange the linear, and substitute into the quadratic

$$x = 10 - 3y$$

$$\text{so } (10 - 3y)^2 + y^2 = 20$$

$$\text{Expand and solve the quadratic}$$

$$100 - 60y + 9y^2 + y^2 = 20$$

$$10y^2 - 60y + 80 = 0$$

$$y = 2 \text{ or } y = 4$$

Finally, substitute into the linear and solve, pairing values...

$$x + 3 \times 2 = 10 \text{ so } (x, y) = (4, 2)$$

$$x + 3 \times 4 = 10 \text{ so } (x, y) = (-2, 4)$$

Sequences A24, A25

n th term of an arithmetic (linear) sequence is $bn + c$

→ n th term of 5, 8, 11, 14, ... is $3n + 2$ (always increases by 3; first term is $3 \times 1 + 2 = 5$)

n th term of a quadratic sequence is $an^2 + bn + c$

→ First three terms of $n^2 + 3n - 1$ are 3, 9, 17, ...

Geometric sequence; multiply each term by a constant ratio

→ 3, 6, 12, 24, ... (ratio is 2)

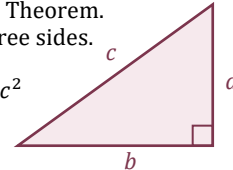
Fibonacci sequence; make the next term by adding the previous two ...

→ 2, 4, 6, 10, 16, 26, 42, ...

Right angled triangles

Pythagoras Theorem. Links all three sides. No angles.

$$a^2 + b^2 = c^2$$



The longest side of any right angled triangle is the hypotenuse; check that your answer is consistent with this.

Advanced trigonometry G21, G22

Sine Rule

Use if you are given an angle-side pair

$$\text{Missing side: } \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\text{Missing angle: } \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

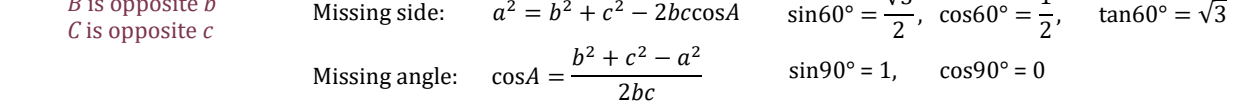
Cosine Rule

Use if you can't use the sine rule

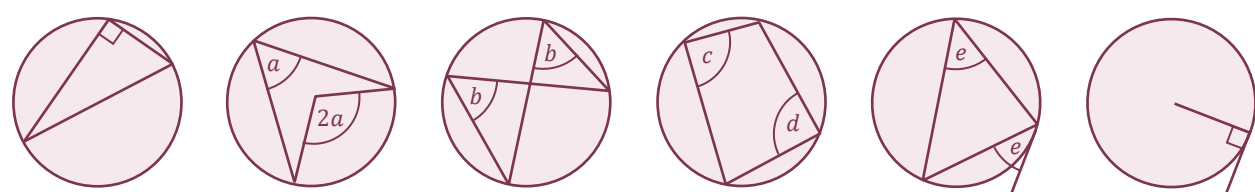
$$\text{Missing side: } a^2 = b^2 + c^2 - 2bccosA$$

$$\text{Missing angle: } cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

Circle theorems



Angle in a semicircle is 90° Angle at the centre is double the angle at the circumference Angles in the same segment are equal Opposite angles in a cyclic quadrilateral total 180° Alternate segment theorem Tangent and radius are perpendicular



Circumference of circle = $\pi \times D$ Area of circle = $\pi \times r^2$ Area of triangle = $\frac{1}{2}absinC$ Area of trapezium = $\frac{1}{2}(a + b) \times h$ Volume of prism = area of cross section \times length Volume of cone = $\frac{1}{3}\pi r^2 h$ Volume of frustum is difference between the volumes of two cones

Transformations G7, G8

Reflection: Line of reflection, Vector

Rotation: Centre of rotation, Angle of rotation, Clockwise or anticlockwise

Enlargement: Centre of enlargement, Scale factor (if $-1 < SF < 1$ the shape will get smaller).

Similar shapes: Ratios in similar shapes and solids: Length/perimeter $1:n$ $a:b$, Area $1:n^2$ $a^2:b^2$, Volume $1:n^3$ $a^3:b^3$

Iteration A20

You will be given the formula to use:

→ Solve $x^3 + 6x + 4 = 0$ by using the iteration $x_{n+1} = \sqrt[3]{6x_n - 4}$.

Start with $x_1 = -2.8$.

$$x_2 = \sqrt[3]{6 \times (-2.8) - 4} = -2.750 \dots$$

$$x_3 = \sqrt[3]{6 \times (-2.750 \dots) - 4} = \dots$$

Repeat until you know the solution, or you do as many as the question says.

Percentages: multipliers R9, R16

Percentage increase or decrease; use a multiplier (powers for repetition)

→ Initially there were 20 000 fish in a lake. The number decreases by 15% each year. Estimate the number of fish after 6 years.

$$20\,000 \times 0.85^6 = 7\,500 \text{ (2sf)}$$

Formula for compound interest

$$\text{Total accrued} = P \left(1 + \frac{r}{100}\right)^n$$

→ I invest £600 at 3% compound interest. What is my account worth after 5 years?

$$£600 \times \left(1 + \frac{3}{100}\right)^5 = £695.56$$

Direct & inverse proportion R10

y is directly proportional to x : $y = kx$ for a constant k

→ b is directly proportional to a^2 ; $a = 6$ when $b = 90$. Find b if $a = 8$.

$$b = ka^2; a = 6 \text{ and } b = 90 \text{ for } k; 90 = k \times 6^2 \text{ so } k = 2.5; b = 2.5a^2$$

$$b = 2.5 \times 8^2 = 160$$

y is inversely proportional to x : $yx = k$ or $y = \frac{k}{x}$ for a constant k

Probability rules P8, P9

Multiply for independent events

→ P(6 on dice and H on coin)

$$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$$

Add for mutually exclusive events

→ P(5 or 6 on dice)

$$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$$

Apply these rules to tree diagrams.

Histograms S3

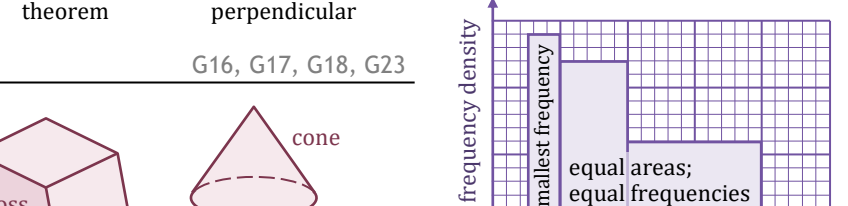
In general... $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$$P(A \text{ and } B) = P(A \text{ given } B) \times P(B)$$

Box plots S4

Frequency = frequency density multiplied by class width. This means that bars with the same frequency have the same area.

Interquartile range (IQR) = UQ - LQ



Box plots S4

Interquartile range (IQR) = UQ - LQ

